PEDAGOGY OF MATHEMATICS

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Chapter: 1:

Meaning, Nature and Scope of Mathematics.

Meaning of Mathematics:

Mathematics is the science of measurement, quantity and magnitude. Mathematics is known as "Ganita" in Hindi which means the science of calculation. Developing children's abilities for mathematics is the main goal of mathematics education. The narrow aim of school mathematics is to develop 'useful' capabilities, particularly those relating to numeracy–numbers, number operations, measurements, decimals and percentages. The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems.

Mathematics is the most closely related subject in our daily life. Its knowledge is exact, systematic logical and clear. Mathematics involves the process for intellectual development of mental faculties. It is not that mathematical knowledge is needed only by engineers, doctors or business personals. Even the smallest citizen of society such as laborers and workers need the basic knowledge of mathematics. Besides the mental ability, mathematics develops some quality like concentration, truthfulness, seriousness and reasoning. Thus, in the words of Locke it is rightly said that, **"Mathematics is a way to settle in the mind the habit of reasoning".**

Definition of Mathematics:

"The abstract science which investigates deductively the conclusions implicit in the elementary conceptions of spatial and numerical relations, and which includes as its main divisions geometry, arithmetic, and algebra" - Oxford English Dictionary, 1933

"The study of the measurement, properties, and relationships of quantities and sets, using numbers and symbols" - American Heritage Dictionary, 2000

"The science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the shapes of objects" - Encyclopaedia Britannica

Nature of Mathematics

Based on the various definitions, the nature of mathematics can be listed as follows:

- **Mathematics – A science of discovery:**

Mathematics is the discovery of relationships and the expression of those relationships in symbolic form – in words, in numbers, in letters, by diagrams or by graphs. According to A.N.Whitehead (1912) "Every child should experience the joy of discovery". Initially a child's discoveries may be observational. But later, when its power of abstraction is adequately developed, it will be able to appreciate the certitude of the mathematical conclusions that it has drawn. This will give it the joy of discovering mathematical truths and concepts. Mathematics gives an early opportunity to make independent discoveries.

The children must not only have opportunities for making their own discoveries of mathematical ideas, but they must also have the practice necessary to achieve accuracy in their calculations. Today it is discovery techniques, which are making spectacular progress. They are being applied in two fields: in pure number relationships and in everyday problems, involving such things as money, weights and measures.

- **Mathematics –An intellectual game**

Mathematics can be treated as an intellectual game with its own rules and without any relation to external criteria. From this viewpoint, mathematics is mainly a matter of puzzles, paradoxes, and problem solving – a sort of healthy mental exercise.

Mathematics – The art of drawing conclusions:

One of the important functions of the school is to familiarize children with a mode of thought which helps them in drawing right conclusions and inferences. According to J.W.A. Young a subject suitable for this purpose should have three characteristics:

1. That its conclusion are certain. At first, at least it is essential that the learner should know whether or not he has drawn the correct conclusion.
2. That it permits the learner to begin with simple and very easy conclusions to pass in well graded sequence to very difficult ones, as the earlier ones are mastered
3. That the type of conclusions exemplified in the introductory subject be found in the other subjects also, and in human interactions, in general.

These characteristics are present in mathematics to a larger extend than in any other available subject.

- **Mathematics – A tool subject**

It could be more elegantly expressed as "mathematics, handmaiden to the sciences". From the beginning, down to the nineteenth century, mathematics has been assigned the status of a servant. Then in the nineteen century, mathematics attained independence. It achieved a completeness and internal consistency that it has not known before. Mathematics continued to be useful to other disciplines, but now it is dependent upon none of them. With its new found freedom, mathematics established its own goals to pursue. Its mentors of the past- engineering, physical science and commerce – now became no more than its peers.

Mathematics has its integrity, its beauty, its structure and many other features relate to mathematics as an end in itself. However, many conceive mathematics as a very useful means to other ends, a powerful and incisive tool of wide applicability.

- **Mathematics – A system of logical processes**

Polya suggested that mathematics actually has two faces. One face is a 'systematic deductive science'. This has resulted in presenting mathematics as an axiomatic body of definitions, undefined terms, axioms, and theorems. Mario Pieri stated "Mathematics is a hypthetico-deductive system". This statement means that mathematics is a system of logical processes whereby conclusions are deduced from certain fundamental assumptions and definitions that have been hypothesized. This has been reinforced by Benjamin Pierce when he defined mathematics as 'The science which draws necessary conclusions'. The student draws the inferences from the premises, provided the premises are true. In mathematics, granted the premises, conclusion follows inevitably. For example:

"When two lines intersect, vertically opposite angles are equal"

(the premise) are vertically opposite angles.

Hence are equal (the inference)

- **Mathematics – An intuitive method**

Intitution implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytic apparatus of one's craft. It is the intitutive mode that yields hypothesis quickly. It precedes proof; it is what the techniques of analysis and proof is designed to test and check. It is a form of mathematical activity which depends on the confidence in the applicability of the process rather than upon the importance of right answers all the time.

Intuition when applied to mathematics involves the concretisation of an idea not yet sated in the form of some sort of operations or example. A child forms an internalized set of structures for representing the world around him. These structures are governed by definite rules of their own. In the course of development, these structures change and the rules governing them also change in certain systematic ways. To anticipate what will happen next and what to do about it is to spin our internal models just a bit faster than the world goes. It is important to allow the student to express his intuition and check and verify its validity. When mathematics is taught in a very formal way by stating the logical rules, and algorithm, we remove his confidences in his ability to perform mathematical processes. Teachers quite often provide formal proof (which is necessary for checking) in place of direct intuition. For example, to check the conjecture, 8x is equivalent to 3x+5x, a formal rigorous statement as the following,

"By the commutative principle for multiplication, for every x, 3x+5x=x3+x5. By the distributive principle, for every x, x3+x5=x(3+5). Again by the commutative law, for every x, x(3+5)=(3+5)x or 8x. So for every x, 3x+5x = 8x", could dampen the students' spirit of intuition and interest. It is up to the teacher to allow the child to use his natural and intuitive ways of thinking, by encouraging him to do so and honouring him when he does.

SCOPE OF MATHEMATICS

- In the field of teaching

- In the field of research
- In the field of industry
- In the field of banking and finance
- Application in daily life
- Applicable in science and technology etc.

Chapter: 2

Aims and objective of teaching Mathematics

The aims of teaching and learning mathematics are to encourage and enable students to:

- recognize that mathematics permeates the world around us
- appreciate the usefulness, power and beauty of mathematics
- enjoy mathematics and develop patience and persistence when solving problems
- understand and be able to use the language, symbols and notation of mathematics
- develop mathematical curiosity and use inductive and deductive reasoning when solving problems
- become confident in using mathematics to analyze and solve problems both in school and in real-life situations
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others
- develop a critical appreciation of the use of information and communication technology in mathematics
- Appreciate the international dimension of mathematics and its multicultural and historical perspectives.

Aims of the Secondary School Mathematics Education:
The secondary school mathematics curriculum continues the development of the learning of mathematics in the primary school. To enable students to cope confidently with the mathematics needed in their future studies, workplaces or daily life in a technological and information-rich society, the curriculum aims at developing students: the ability to conceptualize, inquire, reason and communicate mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts; the ability to manipulate numbers, symbols and other mathematical objects; the number sense, symbol sense, spatial sense and a sense of measurement as well as the capability in appreciating structures and patterns; a positive attitude towards mathematics and the capability in appreciating the aesthetic nature and cultural aspect of mathematics.

Objectives:

1. **Knowledge and understanding Domain :**

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop problem-solving skills. Through knowledge and understanding students develop mathematical reasoning to make deductions and solve problems.

To induce children to understand and grasp the knowledge of the following:
—the directed numbers and the real number system the algebraic symbols to describe relations among quantities and number patterns; the equations, inequalities, identities, formulas and functions; the measures for simple 2-D and 3-D figures; the intuitive, deductive and analytic approach to study geometric figures; the trigonometric ratios and functions; the statistical methods and statistical measures; the simple ideas of probability and laws of probability.

2. **Attitude Domain:**
   To foster the attitudes to: be interested in learning mathematics; be confident in their abilities to do mathematics; willingly apply mathematical knowledge; appreciate that mathematics is a dynamic field with its roots in many cultures; appreciate the precise and aesthetic aspect of mathematics; appreciate the role of mathematics in human affairs; be willing to persist in solving problems; be willing to work cooperatively with people and to value the contribution of others.

3. **Skill Domain:**

To develop the following skills and capabilities in: basic computations in real numbers and symbols and an ability to judge reasonableness of results; using the mathematical language to communicate ideas; reasoning mathematically, i.e. they should conjecture, test and build arguments about the validity of a proposition; applying mathematical knowledge to solve a variety of problems; handling data and generating information; number sense and spatial sense; using modern technology appropriately to learn and do mathematics; learning mathematics independently and collaboratively for the whole life.

4. **Investigating patterns**

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Mathematical inquiry encourages students to become risk-takers,

inquirers and critical thinkers. Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate and/or analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

5. Communication in mathematics

Mathematics provides a powerful and universal language. Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings—both orally and in writing.

Students are encouraged to choose and use ICT tools as appropriate and, where available, to enhance communication of their mathematical ideas. ICT tools can include graphic display calculators, screenshots, graphing, spreadsheets, databases, and drawing and word-processing software.

6. Reflection in mathematics

Mathematics encourages students to reflect upon their findings and problem-solving processes. Students are encouraged to share their thinking with teachers and peers and to examine different problem-solving strategies. Critical reflection in mathematics helps students gain insight into their strengths and weaknesses as learners and to appreciate the value of errors as powerful motivators to enhance learning and understanding.

Chapter: 3

Values of Teaching Mathematics

The Value of Mathematics is used in each step for our life. We cannot survive without Mathematics. It gives happiness in our life when we make our career with help of Mathematics.

1. **Practical or Utilitarian Value :** Everybody uses mathematics in their daily life. Any person who is ignorant of mathematics can be easily cheated. He will always be at the mercy of others. We have to make purchases daily. We buy cloth, food items, fruit, vegetables, grocery etc. We have to calculate how much we have to pay for everything. Mathematics is needed by everyone whether rich or poor, high or low.
2. **Intellectual values :** Develops intellectual powers , imagination , Memorization ,Observation , Invention , Concentration , Originality , Creativity , reasoning . If a child faces mathematical problems, his mind become active in solving that problem.
3. **Social values :** Social values can be acquired like: Tolerance ,Open mindedness ,Objectivity , Honesty , Truthfulness , Co-operation , Will power, Organization and maintenance of social institutions , banks , co-operations ,Railways . post offices
4. **Moral values :** Ability in arguing on the correctness and incorrectness of a statement. Develops moral qualities, Honesty , Truthfulness , Justice ,Dutifulness ,Punctuality , self confidence , power of distinguishing between right and wrong , Cleanness , Patience , listening to others , respect to others Deprives off the feelings of jealousy, hate etc.

5. **Disciplinary Values** : "Mathematics is the way to settle in the mind a habit of reasoning" -Locke . As mathematical knowledge is exact, logical, real and to that point, it creates discipline in a human mind. Develops an ability to grasp a situation , to analyse the situation and to perceive correctly the state of affairs .Helps the students to imbibe qualities, Simplicity, Accuracy , Certainty of results , Originality , Similarity to reasoning in life ,Verification of results , Concentration of mind .
6. **Cultural values** : "Mathematics is the mirror of civilization" By the culture of a nation or society we mean the mode of living of its inhabitants. Helps in promoting cultural heritage an transmitting it to future generation. The welfare of our civilization is almost depends upon scientific and mathematical developments. Helped in the development of various subjects and occupations.
7. **International values**: Mathematicians research their mathematical ideas from one nation to another nation. Mathematician's books and journals are circulated among almost all the nations of the world.
8. **Aesthetic values :** "Education as a whole should foster the higher impersonal pleasures" -Thorndike One gets pleasure in solving mathematical problems, especially when he get correct answers to his problems. Different symmetric designs by Ramanujan are a source of great pleasure Eg: Mathematics is closely related with arts like drawing, painting, music etc .All musical instruments like harmonium, drum, flute, guitar, violin etc are played with the rules of mathematics.
9. **Vocational values :** Mathematics has great vocational values. Study of mathematics prepares the students for a wide variety of vocations. It finds extensive application in all vocations like: Agriculture , Accountancy , Banking , Business , Engineering , information technology , Tailoring , Carpentry . Surveying etc.
10. **Psychological values :** Mathematics helps to develop positive attitudes such as open mindedness, reasoning etc. The learning of mathematics is based on fundamental principles of psychology such as learning by doing learning through experiences and problem solving etc.

Teaching of mathematics results in the development of a number of fruitful values in the students. Only a resourceful teacher of mathematics with his deliberate efforts and planning can make it possible for the students to realize these values. So we have to realise the value & product good civilian for our nation and for the betterment of society.

Chapter: 4

Relation of Mathematics with other School subjects:

Mathematics and Economics

The level of mathematical literacy required for personal and social activities is continually increasing. Mastery of the fundamental processes is necessary for clear thinking. The social sciences are also beginning to draw heavily upon mathematics.

Mathematical language and methods are used frequently in describing economic phenomena. According to Marshall – "The direct application of mathematical reasoning to the

discovery of economic truths has recently rendered great services in the hand of master mathematicians." Statistical methods are used in economic forecast different issues of economics can be represented statistically such as 'Trade Cycles', Volume trade, trend of exports and imports, population trends, industrial trends, thrift, expenditure of public money etc.,

In economic theory and econometrics, a great deal of mathematical work is being done all over the world. In econometrics, tools of matrices, probability and statistics are used. A great deal of mathematical thinking goes in the task of national economic planning, and a number of mathematical models for planning have been developed.

Mathematics in Political Science

In Mathematical Political Science, we analyze past election results to see changes in voting patterns and the influence of various factors on voting behavior, on switching of votes among political parties and mathematical models for Conflict Resolution. Here we make use of Game Theory.

Mathematics in Geography

Geography is nothing but a scientific and mathematical description of our earth in its universe. The dimension and magnitude of earth, its situation and position in the universe the formation of days and nights, lunar and solar eclipses, latitude and longitude, maximum and minimum rainfall, etc are some of the numerous learning areas of geography which need the application of mathematics. The surveying instruments in geography have to be mathematically accurate. There are changes in the fertility of the soil, changes in the distribution of forests, changes in ecology etc., which have to be mathematically determined, in order to exercise desirable control over them.

Mathematics in History:

- **To know the time period**
- **To know the birth and death of historical persons**

in history Mathematics helps in Calculation of Dates like duration of Britishers ruled in India? When Gandhi ji was born? Celebrate National Days and festivals, Cost in building of Taj-Mahal. Tenure of President in India. This gives us new information of the historical world. When the First and second world wars were fought? On account of economic considerations industrial revolution in Europe.

- **MATHEMATICS WITH PHYSICAL EDUCATION:** Mathematics is used to measure structure of the body, blood pressure, the height, weight, rules of the games

etc. Temperature of the normal human body, Size of playground, norm and standard of game like boot-ball, hockey, cricket, volleyball, tennis, wrestling, boxing etc.

- **MATHEMATICS WITH COMMERCE/ACCOUNTS:** With the rich knowledge of commerce it is possible to study the economy of the country. Only by the knowledge of mathematics, Debit, Credit process & expenditure in accounts of industry, banks firm, etc are determined. The commerce teacher of should try to teachor make understand in such a way that students may relate and explain all specific terms mathematically.
- **MATHEMATICS AND FINE ARTS**: decides size, Ratio and Proportion while constructing the Similarity, Scale appreciation, Balance and Symmetry, Postulates, Drawing images on cloth and paper, Rhythm in Music etc.
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MATHEMATICS AND LANGUAGE:

- *Math and Reading*:- Students read about the discoveries or work of great mathematicians, and they can make poem on numbers.
- *Math and Writing* (numbers are converted into writing):- A student makes the pie chart and interprets in his own words.

e.g. Counting of alphabet, vowel, Read About The Life History of Mathematicians. Student can draw make a bar graph of time spent in school and home the whole week and can interpret. (Interpretation of Non-Verbal Data)

Chapter: 5

Recommendation of NCF-2005 on teaching Mathematics.

Developing children's abilities for mathematisation is the main goal of mathematics education. The narrow aim of school mathematics is to develop 'useful' capabilities, particularly those relating to numeracy–numbers, number operations, measurements, decimals and percentages. The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems. This calls for a curriculum that is ambitious, coherent and teaches important principles of mathematics. It should be ambitious in the sense that it seeks to achieve the higher aim mentioned above, rather than only the narrower aim. It should be coherent in the sense that the variety of methods and skills available piecemeal (in arithmetic, algebra, geometry) cohere into an ability to address problems that come from other domains such as science and social studies in high school. It should be important in the sense that students feel the need to solve such problems, that teachers and students find it worth their time and energy to address these problems.

As mathematics is a compulsory subject at the secondary stage, access to quality mathematics education is the right of every child. Most of the skills taught in primary school mathematics are useful. However, a reorientation of the curriculum towards addressing the 'higher aims'

mentioned above will make better use of the time that children spend in school in terms of the problem-solving and analytical skills that it builds, and in preparing children to better meet a wide variety of problems in life.

Vision for School Mathematics

• Children learn to enjoy mathematics rather than fear it.

• Children learn important mathematics: Mathematics is more than formulas and Mechanical procedures.

• Children see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on. Children pose and solve meaningful problems.

• Children use abstractions to perceive relation-ships, to see structures, to reason out things, to argue the truth or falsity of statements.

• Children understand the basic structure of Mathematics: Arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalisation.

• Teachers engage every child in class with the conviction that everyone can learn mathematics.

The Curriculum:

At the pre-primary stage, all learning occurs through play rather than through didactic communication. Rather than the rote learning of the number sequence, children need to learn and understand, in the context of small sets, the connection between word games and counting, and between counting and quantity. Making simple comparisons and classifications along one dimension at a time, and identifying shapes and symmetries, are appropriate skills to acquire at this stage. Encouraging children to use language to freely express one's thoughts and emotions, rather than in predetermined ways, is extremely important at this and at later stages.

At the primary stage : Having children develop a positive attitude towards, and a liking for, Mathematics at the primary stage is as important, if not more than the cognitive skills and concepts that they acquire. Mathematical games, puzzles and stories help in developing a positive attitude and in making connections between mathematics and everyday thinking. It is important to note that mathematics is not just arithmetic. Besides numbers and number operations, due importance must be given to shapes, spatial understanding, patterns, measurement and data handling. The curriculum must explicitly incorporate the progression that learners make from the concrete to the abstract while acquiring concepts. Apart from computational skills, stress must be laid on identifying, expressing and explaining patterns, on estimation and approximation in solving problems, on making connections, and on the development of skills of language in communication and reasoning.

At the upper primary stage, students get the first taste of the power of Mathematics through the application of powerful abstract concepts that compress previous learning and experience. This enables them to revisit and consolidate basic concepts and skills learnt at the primary stage, which is essential from the point of view of achieving universal mathematical literacy.

Students are introduced to algebraic notation and its use in solving problems and in generalisation, to the systematic study of space and shapes, and for consolidating their knowledge of measurement. Data handling, representation and interpretation form a significant part of the ability of dealing with information in general, which is an essential 'life skill'. The learning at this stage also offers an opportunity to enrich students' spatial reasoning and visualisation skills.

At the secondary stage, students begin to perceive the structure of Mathematics as a discipline. They become familiar with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions, and proofs justifying propositions. These aspects are developed particularly in the area of geometry. Students develop their facility with algebra, which is important not only in the application of mathematics, but also within mathematics in providing justifications and proofs. At this stage, students integrate the many concepts and skills that they have learnt into a problem-solving ability. Mathematical modelling, data analysis and interpretation taught at this stage can consolidate a high level of mathematical literacy. Individual and group exploration of connections and patterns, visualisation and generalisation, and making and proving conjectures are important at this stage, and can be encouraged through the use of appropriate tools that include concrete models as in Mathematics laboratories and computers.

At the higher secondary stage : The aim of the Mathematics curriculum at the higher secondary stage is to provide students with an appreciation of the wide variety of the application of Mathematics, and equip them with the basic tools that enable such application. A careful choice between the often conflicting demands of depth versus breadth needs to be made at this stage. The rapid explosion of Mathematics as a discipline, and of its range of application, favours an increase in the breadth of coverage. Such increase must be dictated by mathematical considerations of the importance of topics to be included. Topics that are more naturally the province of other disciplines may be left out of the Mathematics curriculum. The treatment of topics must have an objective, that is, the communication of mathematical insights and concepts, which naturally arouse the interest and curiosity of students.

Instructional Method of Teaching Mathematics.

Chapter: 6
Method of teaching Mathematics

Inductive-Deductive Method

Inductive Method:

It leads concrete to abstract, particular to general, observation to theory and example to formula.

A Inductive approach to instruction is a more child -centred approach

Procedure: First do lots of example, and then generalize the formula.

Example: Ask students to draw a few sets of parallel lines and let them measure the alternate angle.

Ask student to construct the few triangles. Let them measure the sum of the angles and draw conclusion.

A Child Observes a rising of sun and getting of darkness after the setting of sun

Conclusion: "The Sun rises every day and also Sets everyday"

A child eats green apple every time and feels its sour taste.

Conclusion: all the green apples are sour in taste

Merits of Inductive Method

By making use of this method, following merits get accrue to the students as well as to teacher:

1. As this is a scientific method, thus it helps to considerable extent in developing scientific outlook among the students.
2. This method helps to develop scientific attitude among the students.
3. With the help of this method, teacher can develop qualities of critical thinking and habit of keen observation among the students properly and accurately.
4. This is a very logical and psychological kind of teaching science.
5. By this method, students get various opportunities to play an active role in learning process.

Demerits of Inductive Method

This method has certain limitations, some of which are as follows:

1. The results or conclusions drawn from such method are not found to be final in case where the amount of data is very large in number.
2. All the topics of science cannot be dealt with this method properly.
3. This method can only be used when teacher have much time for teaching process.

Deductive method:

Opposite of inductive method. Here, the learner proceeds abstract to concrete, general to particular, , theory to observation and formula to examples.

A deductive approach to instruction is a more teacher-centred approach

Procedure: immediately after announcing the topic for the day, the teacher gives the relevant formula and solves some problem related to formula. The student understands how the formula can be used or applies. Then few problems are given to the student to solve by themselves.

Example: Facts like sum of the angle is 180^0 and solve the problem related to given facts.

Formula like area of rectangle = Length x Breadth and solve problem related to given formula.

A child is told “the sun rises every day and also sets every day!” this fact child verifies by daily observation

“All the green apples are sour in taste” the child may be told that he should never eat the green apple because they are sour. Afterwards he may verify this facts by tasting green apples.

Merits

(1) This method is short and time-saving. The solution of the problems by pre-established formulas takes little time.

(2) It encourages memory as the students have to memories a considerable number of formulas.

(3) This method is advantageous at the “Practice and revision” stage.

(4) It enhances speed and efficiency in solving problems.

(5)This removes the incompleteness and inadequacy of Inductive method.

Demerits

(1) The beginners find it very difficult to understand an abstract formulas, if they are not acquainted with a number of concrete instances.

(2) This method will demand blind memorization of a large number of formulas. And this will cause an unnecessary and heavy burden on the brain of children.

(3) In this method, memory becomes more important than understanding and intelligence and that is educationally unsound.

(4) Blind cramming leads very often to forgetting the formulas and the children are at a loss to recollect. This ultimately leads to no learning

(5) This method is not suitable for development of thinking, reasoning and discovery.

Conclusion: we can conclude that inductive method is the forerunner of deductive method. The deductive method will give a good follow up, if the topic is understand through induction. thus the teaching must begin with induction and end in deduction.

Analytic Method Synthetic Method

Analytic Method:

It proceeds from unknown to known. 'Analysis' means 'breaking up'. In this method we **break up** the unknown problem into simpler parts and then see how these can be recombined to find the solution. So we start with what is to be found out and then think of further steps or possibilities the may connect the unknown built the known and find out the desired result.

Procedure:

Example: If $(ac- 2b^2)\div b = (c^2-2bd) \div d$ prove that $a/b=c/d$,

Solution start from unknown i.e. to solve this problem we have to start from

$(ac- 2b^2)\div b = (c^2-2bd) \div d$ after calculation

We get the answer $a/b=c/d$

Example:

If $a^2+b^2=7ab$ prove that $2\log (a+b) = 2\log3+\log a+\log b$

Proof:

To prove this using analytic method, begin from the unknown.

The unknown is $2\log (a+b) = 2\log3+\log a+\log b$

Now, $2\log (a+b) = 2\log 3+ \log a+ \log b$ is true

If $\log (a+b)^2 = \log 3^2 + \log a + \log b$ is true

If $\log (a+b)^2 = \log 9 + \log ab$ is true

If $\log (a+b)^2 = \log 9ab$ is true

If $(a+b)^2 = 9ab$ is true

If $a^2+b^2+2ab =9ab$ is true

Then $a^2+b^2=7ab$ which is known and true

Thus if $a^2+b^2= 7ab$ prove that $2\log (a+b) = 2\log3+\log a+\log b$

Merit of Analytic Method

- It is logical method. It leaves no doubts and convinces the learner.
- It helps student in understanding and strengthen the urge to discover facts.
- Each steps has the its reason and justification. So, no fixed steps is required.
- It is a method of discovery.
- It is a process of thinking (exploration).
- It pulls apart or breaks up the statement under solution.
- It answers satisfactorily and question that may arise in the mind of pupil.
- The students can recall and reconstruct easily any step if forgotten.
- It develops originality.

Demerit of Analytic Method

- It is lengthy method and laborious.

- It is difficult to acquire efficiency and speed
- It may not be applicable to all topics equally well.
- It is slow, round-about and involves trial and error.

Synthetic Method

Synthesis means building up separate element and their combination.It proceeds from the known to the unknown facts. It is opposite of analytic.

Procedure:

Example: if a/b=c/d, prove that $ac-2b^2/b=c^2-2bd/d$

Solution: start from known i.e. to solve this problem we have to start from

a/b=c/d ; subtract both side by 2b/c

and get the answer $ac-2b^2/b=c^2-2bd/d$. it is unknown

Merit Synthetic Method

- It is short method, it glorify memory, it suit the teachers and it follow the same process as given in the text book.
- It is a method of presentation of discovered facts.
- It is a product of thought.
- It is short and concise.
- It puts together or synthesizes known facts.
- Once forgotten, it cannot be recalled.
- It is quick, straight forward and does without trail and error.

- It does not satisfy doubts and questions arising in the mind of the leaner.
- It is a special device; it is a method for the crammer.
- It is not that easy to recall or reconstruct any forgotten step.
- It develops memory.
- It is informal.
- It is simply informational.
- There is no heuristic approach in it.
- It is the follower of analysis.

Demerits Synthetic Method

- It is not a psychological method.
- There is a scope for forgetting.
- It makes the students passive listeners and encourages cramming.
- In this method confidence is generally lacking in the student.
- There is no scope of discovery.
- The recall of each step cannot be possible for every child.

Difference between Analytic Method & Synthetic Method

No.	**Analytic Method**	**Synthetic Method**
1	Analysis means breaking up into simpler elements.	Synthesis means building up separate element and their combination
2	It proceeds from the unknown to the known facts.	It proceeds from the known to the unknown facts.
3	It is a method of discovery.	It is a method of presentation of discovered facts.
4	It is a process of thinking (exploration).	It is a product of thought.
5	It is lengthy and laborious.	It is short and concise.
6	It pulls apart or breaks up the statement under solution.	It puts together or synthesizes known facts.
7	It can be rediscovered.	Once forgotten, it cannot be recalled.

8	It is slow, round-about and involves trial and error.	It is quick, straight forward and does without trail and error.
9	It answers satisfactorily and question that may arise in the mind of pupil.	It does not satisfy doubts and questions arising in the mind of the leaner.
10	It is a general method; it is a method for the thinker and discoverer.	It is a special device; it is a method for the crammer.
11	The students can recall and reconstruct easily any step if forgotten.	It is not that easy to recall or reconstruct any forgotten step.
12	It develops originality.	It develops memory.
13	It is informal.	It is informal.
14	It is formational.	It is simply informational.
15	It is based on heuristic lines.	There is no heuristic approach in it.
16	It is fore-runner of synthetic.	It is the follower of analysis.

Conclusion: Since analysis is a lengthy method, it needs the help of Synthesis for the removal of doubts. Synthesis is the complement of the analysis method. both the method are interdependent. Teachers should offer help for the analytic form of the solution and that synthetic work should be left for the students.

HEURISTIC METHOD OR DISCOVERY METHOD

Heuristic i.e I find (greek word) The method involves finding out by the students. The teacher's role is to provide timely guidance and supplementary materials and ask thought provoking questions to lead them in right direction. This type of strategy is known as Heuristic Method or Discovery Method.

This method is more important from educational point of view because in this method students work like a researcher and solve the problems. By use of this method , scientific and mathematical attitude can be develop in learners.

Some of the common methods labeled as a project method, problem- solving method , activity method, induction method fall under the discovery method.

MERITS:

1. Active participation
2. Improve the thinking level
3. Home work and memorization work become light
4. Improve the creativity and constructive tendency in the learners
5. Happiness & mental satisfaction and encourages him towards further achievement.
6. It gives understanding of concept and facts.
7. It develops scientific and critical attitude in the students.
8. It develops self-reliant
9. It arouses the spirit of enquiry in the students
10. It is psychological sound system of learning as it is based on the principle of learning by doing.

DEMERIT

1.It is lengthy and time consuming methods.

2. It is method is feasible only with a highly resourceful teacher.

3. Not all students can cope with this type of learning

4 IT demands individual attention on the part of teacher.

5.It is not applicable for all level of class and students.

LABORATARY METHODS

It is based on activity. This activity leads the pupil to discover mathematical facts. It is also based on the principles of learning by doing. Learning by observation and proceeding from concrete to abstract.

Merit:

1. It is interesting and joyful for learner
2. It is based on learning by doing.
3. The learner acquires a clear understanding of the subject.
4. It helps in the growth of self-reliance
5. Some topics of mathematics are best understood through this method.

Demerit:

1. The method is very expensive
2. All the topic of mathematics cannot exclusively be taught by this method
3. It is slow method.
4. Teacher will be required to pay individual attention.
5. It is not at all easy to make the students discover mathematics facts experimentally, especially in lower classes.

Conclusion:

It is a difficult and lengthy method, but can prove exceedingly profitable if properly employed. Young children will then be fascinated by this method. This method should be a must where circumstances favour.

Problem Solving Methods

Mathematics is an essential discipline because of its practical role to the individual and society. Through a problem-solving approach, this aspect of mathematics can be developed. Presenting a problem and developing the skills needed to solve that problem is more motivational than teaching the skills without a context. The problem methods aims at presenting the knowledge to be learnt in the form of a problem.

The child is curious by nature. He wants to find out solutions of many problems, which sometimes are puzzling even to the adults. The problem solving method is one, which involves the use of the process of problem solving or reflective thinking or reasoning. Problem solving method, as the name indicated, begins with the statement of a problem that challenges the students to find a solution.

Procedure for Problem solving:

1. Identifying and defining the problem:

The student should be able to identify and clearly define the problem. The problem that has been identified should be interesting challenging and motivating for the students to participate in exploring.

2. Analysing the problem:

The problem should be carefully analysed as to what is given and what is to be find out. Given facts must be identified and expressed, if necessary in symbolic form.

3. Formulating tentative hypothesis

Formulating of hypothesis means preparation of a list of possible reasons of the occurrence of the problem. Formulating of hypothesis develops thinking and reasoning powers of the child. The focus at this stage is on hypothesizing – searching for the tentative solution to the problem.

4. Testing the hypothesis:

Appropriate methods should be selected to test the validity of the tentative hypothesis as a solution to the problem. If it is not proved to be the solution, the students are asked to formulate alternate hypothesis and proceed.

5. Verifying of the result or checking the result:

No conclusion should be accepted without being properly verified. At this step the students are asked to determine their results and substantiate the expected solution. The students should be able to make generalisations and apply it to their daily life.

Example :

Define union of three sets. If A={2,3,5}. B={3,5,6} And C={4,6,8,9}.

Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

Solution :

Step 1: Identifying and Defining the Problem

After selecting and understanding the problem the child will be able to define the problem in his own words that

(i) The union of two sets A and B is the set, which contains all the members of a set A and all the members of a set B.

(ii) The union of two set A and B is express as 'A ∪ B' and symbolically represented as $A \cup B = \{x ; x \in A \text{ or } x \in B\}$

(iii) The common elements are taken only once in the union of two sets

Step 2: Analysing the Problem

After defining the problem in his own words, the child will analyse the given problem that how the problem can be solved?

Step 3 : Formulating Tentative Hypothesis

After analysing the various aspects of the problem he will be able to make hypothesis that first of all he should calculate the union of sets B and C i.e. (B ∪ C). Then the union of set A and B ∪ C. thus he can get the value of

A ∪ (B ∪ C). Similarly he can solve (A ∪ B) ∪ C

Step 4: Testing Hypothesis

Thus on the basis of given data, the child will be able to solve the problem in the following manner

In the example it is given that

$B \cup C = \{3,5,6\} \cup \{4,6,8,9\}$

$= \{3,4,5,6,8,9\}$

$A \cup (B \cup C) = \{2,3,5\} \cup \{3,4,5,6,8,9\}$

$= \{2,3,4,5,6,8,9\}$

Similarly,

$A \cup B = \{2,3,5,6\}$

$(A \cup B) \cup C = \{2,3,4,5,6,8,9\}$

After solving the problem the child will analyse the result on the basis of given data and verify his hypothesis whether $A \cup (B \cup C)$ is equals to

$(A \cup B) \cup C$ or not.

Step 5 : Verifying of the result

After testing and verifying his hypothesis the child will be able to conclude that $A \cup (B \cup C) = (A \cup B) \cup C$

Thus the child generalises the results and apply his knowledge in new situations.

Merits

1. This method is psychological and scientific in nature
2. It helps in developing good study habits and reasoning powers.
3. It helps to improve and apply knowledge and experience.
4. This method stimulates thinking of the child
5. It helps to develop the power of expression of the child.
6. The child learns how to act in new situation.
7. It develops group feeling while working together.
8. Teachers become familiar with his pupils.
9. It develops analytical, critical and generalization abilities of the child.
10. This method helps in maintaining discipline in the class.

Demerits

1. This is not suitable for lower classes
2. There is lack of suitable books and references for children.
3. It takes more time and energy.
4. Teachers find it difficult to cover the prescribed syllabus.
5. To follow this method talented teacher are required.

Project method

Project method provides a practical approach to learning. This method may be tried along with formal classroom teaching without disturbing the school timetable. This method leads to understanding and develops the ability to apply knowledge.

Project method is of American origin and is an outcome of Dewey's philosophy or pragmatism. However, this method is developed and advocated by Dr.Kilpatrick.

Basic principles of project method:

Psychological principles of learning

1. Learning by doing
2. Learning by living
3. Children learn better through association, co-operation and activity.

Psychological laws of learning

1. Law of readiness
2. Law of exercise
3. Law of effect

STEPS INVOLVED IN PROJECT METHOD

1) Providing / creating the situations
2) Proposing and choosing the project
3) Planning the project
4) Execution of the project
5) Evaluation of the project
6) Recording of the project.

Step 1. **Creating the situation:** The teacher creates problematic situation in front of students while creating the appropriate situation student's interest and abilities should be given due importance.

step 2. **Proposing and choosing the project:** while choosing a problem teacher should stimulate discussions by making suggestions. The proposed project should be according to the rear need of students. The purpose of the project should be well defined and understood by the children.

step 3. **Planning the project:** for the success of the project, planning of project is very import. The children should plan out the project under the guidance of their teacher.

step 4. **Execution of the project:** every child should contribute actively in the execution of the project. It is the longest step in the project.

step 5. **Evaluation of the project:** when the project is completed the teacher and the children should evaluate it jointly discussed whether the objectives of the project have been achieved or not.

step 6. **Recording of the project:** the children maintain a complete record of the project work. While recording the project some points like how the project was planned, what discussion were made, how duties were assigned, hot it was evaluate etc. should be kept in mind.

Some projects for mathematics:

A few projects suitable for high school mathematics are listed below

A. Execution of school bank.
B. Running stationary stores in the school.
C. Planning and estimating the construction of a house
D. Planning for an annual camp
E. Executing the activities of mathematics clubs
F. Collection of data regarding population, death rate, birth rate etc.

Merits:

1. This is based on various psychological laws and principles.
2. It develops self-confidence and self-discipline among the students
3. It provides ample scope for training.
4. It provides score for independent work and individual development.
5. It promotes habits of critical thinking and encourages the students to adopt problem-solving methods.
6. This method the children are active participants in the learning task.
7. This is based on principle of activity, reality, effect, and learning by doing etc.
8. It develops discovery attitude in the child.
9. It provides self-motivation as the students themselves select plan and execute the project.
10. It develops helping attitude in learner.

Demerits :

1. It is very difficult to complete the whole syllabus by the use of this method.
2. Textbooks and instructional materials are hardly available.
3. The project method does not provide necessary drill and practice for the learners of the subject.
4. The project method is uneconomical in terms of time and is not possible to fit into the regular time table.
5. This method is not suitable for a fixed curriculum.

Chapter: 7

Constructivist approach of teaching Mathematics.

Constructivism:

Most traditional mathematics instruction and curricula are based on the *transmission,* or *absorption,* view of teaching and learning. In this view, students passively "absorb" mathematical structures invented by others and recorded in texts or known by authoritative adults. Teaching consists of transmitting sets of established facts, skills, and concepts to students.

Constructivism offers a sharp contrast to this view. Its basic tenets -- which are embraced to a greater or lesser extent by different proponents are the following:

1. Knowledge is actively created or invented by the child, not passively received from the environment. This idea can be illustrated by the Piagetian position that mathematical ideas are *made* by children, not found like a pebble or accepted from others like a gift. For example, the idea "four" cannot be directly detected by a child's senses. It is a relation that the child superimposes on a set of objects. This relation is constructed by the child by reflecting on actions performed on numerous sets of objects, such as contrasting the counting of sets having four units with the counting of sets having three and five units. Although a teacher may have demonstrated and numerically labeled many sets of objects for the student, the mental entity "four" can be created only by the student's thought. In other words, students do not "discover" the way the world works like Columbus found a new continent. Rather they *invent* new ways of thinking about the world.

2. Children create new mathematical knowledge by reflecting on their physical and mental actions. Ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge.

3. No one true reality exists, only individual interpretations of the world. These interpretations are shaped by experience and social interactions. Thus, learning mathematics

should be thought of as a process of adapting to and organizing one's quantitative world, not discovering preexisting ideas imposed by others

4. Learning is a social process in which children grow into the intellectual life of those around them (Burner 1986). Mathematical ideas and truths, both in use and in meaning, are cooperatively established by the members of a culture. Thus, the constructivist classroom is seen as a culture in which students are involved not only in discovery and invention but in a social discourse involving explanation, negotiation, sharing, and evaluation.

5. When a teacher demands that students use set mathematical methods, the sense-making activity of students is seriously curtailed. Students tend to mimic the methods by rote so that they can appear to achieve the teacher's goals. Their beliefs about the nature of mathematics change from viewing mathematics as sense making to viewing it as learning set procedures that make little sense.

Two Major Goals:
Although it has many different interpretations, taking a constructivist perspective appears to imply two major goals for mathematics instruction .

1. students should develop mathematical structures that are more complex, abstract, and powerful than the ones they currently possess so that they are increasingly capable of solving a wide variety of meaningful problems.
2. students should become autonomous and self-motivated in their mathematical activity. Such students believe that mathematics is a way of thinking about problems. They believe that they do not "get" mathematical knowledge from their teacher so much as from their own explorations, thinking, and participation in discussions. They see their responsibility in the mathematics classroom not so much as completing assigned tasks but as making sense of, and communicating about, mathematics. Such independent students have the sense of themselves as controlling and creating mathematics.

Teaching and Learning:
Constructivist instruction, on the one hand, gives preeminent value to the development of students' personal mathematical ideas. Traditional instruction, on the other hand, values only established mathematical techniques and concepts. For example, even though many teachers consistently use concrete materials to introduce ideas, they use them only for an introduction; the goal is to get to the abstract, symbolic, established mathematics. Inadvertently, students' intuitive thinking about what is meaningful to them is devalued. They come to feel that their intuitive ideas and methods are not related to *real* mathematics. In contrast, in constructivist instruction, students are encouraged to use their own methods for solving problems. They are not asked to adopt someone else's thinking but encouraged to refine their own. Although the teacher presents tasks that promote the invention or adoption of more sophisticated techniques, all methods are valued and supported. Through interaction with mathematical tasks and other students, the student's own intuitive mathematical thinking gradually becomes more abstract and powerful.

Because the role of the constructivist teacher is to guide and support students' invention of viable mathematical ideas rather than transmit "correct" adult ways of doing mathematics, some see the constructivist approach as inefficient, free-for-all discovery. In fact, even in its least directive form, the guidance of the teacher is the feature that distinguishes

constructivism from unguided discovery. The constructivist teacher, by offering appropriate tasks and opportunities for dialogue, guides the focus of students' attention, thus unobtrusively directing their learning (Bruner 1986).

Constructivist teachers must be able to pose tasks that bring about appropriate conceptual reorganizations in students. This approach requires knowledge of both the normal developmental sequence in which students learn specific mathematical ideas and the current individual structures of students in the class. Such teachers must also be skilled in structuring the intellectual and social climate of the classroom so that students discuss, reflect on, and make sense of these tasks.

Conclusion:

Constructivist says that how students think about particular mathematical ideas and how instructional environments can be structured to cause students to develop more powerful thinking about those ideas. How might your teaching and classroom environment change if you accept that students must construct their own knowledge? Are the implications different for students of different ages? How do you deal with individual differences? Most importantly, what instructional methods are consistent with a constructivist view of learning?

Chapter: 8

Teaching different part of Mathematics- Arithmetic, Algebra & Geometry.

Arithmetic:

Arithmetic: **It** is one among the oldest and elementary branches of mathematics, originating from the Greek word ‘arithmos’ means number. It deals with numbers and the basic operations- addition, subtraction, multiplication, and division, between them.

Components of Arithmetic:

1. Number system(Even & Odd Numbers, Whole Number Natural Number etc)
2. Multiplication and division
3. Decimals
4. Comparing Fractions
5. Percentages
6. Ratios and Proportion
7. Like and Unlike Fractions
8. Surds
9. Sum of n terms
10. Integers
11. Geometric Sequence
12. Fundamental Theorem,
13. Fraction to decimal
14. Word problems on Decimals, Integers etc.

Arithmetic operations:

The basic operations under arithmetic are addition and subtraction, division and multiplication although the subject involves many other modified operations.

Addition (+)

Addition is among the basic operations in arithmetic. In simple forms, addition combines two or more values into a single term, for example: 2 + 5 = 7, 6 + 2 = 8.

The procedure of adding more than two values is called summation and involves methods to add n number of values.

The identity element of addition is 0, which means that adding 0 to any value gives the same result. The inverse element of addition is the opposite of any value, which means that adding opposite of any digit to the digit itself gives the additive identity. For instance, the opposite of 5 is -5, therefore 5 + (-5) = 0.

Subtraction (−)

Subtraction can be labelled as the inverse of addition. It computes the difference between two values, i.e, the minuend minus the subtrahend. If the minuend is greater than the subtrahend, the difference is positive. If the minuend is less than the subtrahend, the result is negative, and 0 if the numbers are equal.

Multiplication (×)

Multiplication also combines two values like addition and subtraction into a single value or the product. The two original values are known as the multiplicand and the multiplier, or simply both as factors.

The product of a and b is expressed as a·b or a x b. In software languages wherein only characters are used that are found in keyboards, it is often expressed as, a*b (* is called asterisk).

Division (÷)

Division is the inverse of multiplication. It computes the quotient of two numbers, the dividend that is divided by the divisor. The quotient is more than 1 if the dividend is greater than divisor for any well-defined positive number, else it is smaller than 1.

Arithmetic Problems

Question 1: The sum of two numbers is 50 and their difference is 30. Find the numbers.

Solution: Let the numbers be x and y. Now, as per the given situation,

x + y = 50……(i)

and x – y = 30……(ii)

We can write, x = 50-y, from eq.(i),

Therefore, putting the value of x in eq(ii), we get,

50 – y – y = 30

50 -2y = 30

2y = 50-30= 20

y = 20/2 = 10

and x = 50 – y = 50-10 =40

Therefore, the two numbers are 40 and 10.

Question 2: Solve 25+5(27÷3)-9

Solution: 25 + 5(27 ÷ 3) – 9

= 25 + 5(9) – 9 = 25 + 45 – 9 =70 – 9 = 61

Concept of 70% of 30 is 21

70 is the percent.; 30 is the base ; 21 is the part.

To determine the percentage: we have to divide the numerator by denominator and then multiply the resultant to 100.

Percentage formula = (Numerator/Denominator)×100

Example: 2/5 × 100 = 0.4 × 100 = 40 per cent

Question 3: Calculate 10% of 80.

10% of 80 = 10/100 × 80 = 8

Another Methods:

(1) Find the value of 5% of 575
Solution:-
(5×575) ÷100 =2875÷100=28.75
(2) Find the value of 20% of 500
Solution:- 20 × 5 = 100

(3) How to multiply-
(i) 65 by 65 **(ii)** 105 by 105
Solution:-

(i) After 6 number is 7. So 1st multiply 6 by 7,
Means 42 and then put 25. So answer is 4225.
(ii) After 10 is 11. So 1st mutiply10 by 11
Means 110 and then put 25. So answer is 11025.
[NOTE:- We may apply if last digit is 5 with any same number]

Question 4: Suman has a monthly salary of Rs.1200. She spends Rs.280 per month on food. What percent of her monthly salary does she save?

Solution- Suman's monthly salary = Rs.1200

Savings of Suman = Rs. (1200 – 280) = Rs. 920

Fraction of salary she saves = Rs. (920÷1200)

Percentage of salary she saves = (920÷1200) ×100 =76.667 %

Question 5: Expand the number 12345 in the exponent form.

Solution: The number 12345 can be expressed as:

12345 = 1 × 10000 + 2 × 1000 + 3 × 100 + 4 × 10 + 5 × 1

Algebra:

Algebra: It is a kind of arithmetic where we use unknown quantities along with numbers. These unknown quantities are represented by letters of the English alphabet such as X, Y, A, B, etc. or symbols. The use of letters helps us to generalize the formulas and rules that you write and also helps you to find the unknown missing value.

Algebra includes almost everything right from solving elementary equations to the study of the abstractions. Algebra equations are included in many chapters of Maths, which student will learn in their academics. Also, there are a number of formulas and identities present in algebra.

The basics of algebra are: Addition and subtraction of algebraic expressions, Multiplications and division of algebraic expression,
Solving equations Literal equations and formulas

Branches of Algebra:

As it is known that, algebra is the concept based on unknown values called variables. The important concept of algebra is equations. It follows various rules to perform arithmetic operations. The rules are used to make sense of sets of data that involves two or more variables. It is used to analyse many things around us. We will probably use the concept of algebra without realising it. Algebra is divided into different sub-branches such as elementary algebra, advanced algebra, abstract algebra, linear algebra, and commutative algebra.

Elementary Algebra:

Elementary Algebra covers the traditional topics studied in a modern elementary algebra course. Arithmetic includes numbers along with mathematical operations like +, -, x, ÷. But in the field of algebra, the numbers are often represented by the symbols and are called variables such as x, a, n, y. It also allows the common formulation of the laws of arithmetic such as, $a + b = b + a$ and it is the first step that shows the systematic exploration of all the properties of a system of real numbers.

The concepts coming under the elementary algebra includes variables, evaluating expressions and equations, properties of equalities and inequalities, solving the algebraic equations and linear equations having one or two variables, and so on.

Advanced Algebra:

This is the intermediate level Algebra or we can say prerequisite of Elementary Algebra .This algebra has a high level of equations to solve as compared to pre-algebra. Advanced algebra will help you to go through the other parts of algebra such as:

- Equations with inequalities
- Matrices
- Solving system of linear equations
- Graphing of functions and linear equations
- Conic sections
- Polynomial Equation
- Quadratic Functions with inequalities
- Polynomials and expressions with radicals
- Sequences and series
- Rational expressions

Abstract Algebra

Abstract algebra is one of the divisions in algebra which discovers the truths relating to algebraic systems independent of specific nature of some operations. These operations in

specific cases have certain properties. Thus we can conclude some consequences of such properties. Hence this branch of mathematics called abstract algebra.

Abstract algebra deals with algebraic structures like the fields, groups, modules, rings, lattices, vector spaces, etc.

The concepts of the abstract algebra are below-

1. **Sets** – Sets is defined as the collection of the objects that are determined by some specific property for a set. For Example- A set of all the 2 by 2 matrices, the set of two-dimensional vectors present in the plane and different form of finite groups.
2. **Binary Operations** – When the concept of addition is conceptualized, it gives the binary operations. The concept of all the binary operations will be meaningless without a set.
3. **Identity Element** – The numbers 0 and 1 are conceptualized to give the idea of an identity element for a specific operation. Here, 0 is called the identity element for the operation addition, whereas 1 is called the identity element for the operation multiplication.
4. **Inverse Elements** – The idea of Inverse elements comes up with a negative number. For addition, we write "-a" as the inverse of "a" and for the purpose of multiplication the inverse form is written as "a^{-1}".
5. **Associativity** – When integers are added, there is a property known as associativity in which the grouping up of numbers added does not affect the sum. Consider for Example – $(3 + 2) + 4 = 3 + (2 + 4)$

Linear Algebra

Linear algebra is a branch of algebra which applies to both applied as well as pure mathematics. It deals with the linear mappings between the vector spaces. It also deals with the study of planes and lines. It is the study of linear sets of equations with the transformation properties. It is almost used in all the areas of Mathematics. It concerns the linear equations for the linear functions with their representation in vector spaces and through the matrices. The important topics covered in linear algebra are as follows:

- Linear equations
- Vector Spaces
- Relations
- Matrices and matrix decomposition
- Relations and Computations

Commutative algebra

Commutative algebra is one of the branches of algebra that studies the commutative rings and its ideals. The algebraic number theory, as well as the algebraic geometry, depends on the commutative algebra. It includes rings of algebraic integers, polynomial rings, and so on. There are many other areas of mathematics, that draw upon commutative algebra in different ways such as differential topology, invariant theory, order theory, and general topology. It has occupied a remarkable role in modern pure mathematics.

BODMAS RULE:

BODMAS is an acronym and it stands for Bracket, of, Division, Multiplication, Addition and Subtraction. It explains the order of operations to solve an expression. According to BODMAS rule, if an expression contains brackets (), {}, [] we have to first solve or simplify the bracket followed by of (powers and roots etc.), then division, multiplication, addition and subtraction from left to right. Solving the problem in the wrong order will result in a wrong answer.

Check the examples below to have a better understanding of using the BODMAS rule.

Simplify:

$1800 \div 10\{ (12-6)+(24-12) \}$

$=1800 \div 10\{6+12\} = 180\{18\} = 180 \times 18 = 3240$

Important Formulae:

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$
$(x+a)(x+b) = x^2 + (a+b)x + ab$
$(x+a)(x-b) = x^2 + (a-b)x - ab$
$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$(ab)^m = a^m b^m$

Geometry

Geometry is derived from two Latin words, Geo +Metron Meaning is earth & measurement. Thus it is concerned with the properties and relations of points, lines, surfaces, solids, and higher dimensional analogy. It is the most practical branch of mathematics that deals with shapes and sizes of figures and their properties. We study geometry to find the Length, Area, Volume of different Plane and Solid figures which are present around us in this world and to know better about them. Knowledge of Coordinate Geometry provides many fundamental skills and helps us to improve problem-solving skill, logical skill, analytical reasoning skill and so on.

Contents in Geometry: Plane Geometry, Points Lines. Angle ,Polygon, Circle, Solid Geometry, Edges, Faces, Vertices ,etc.

(i) Algebraic Geometry – It is a branch of geometry studying zeros of the multivariate polynomial. It includes linear and polynomial algebraic equation used for solving the sets of zeros. The application of this type includes Cryptography, string theory, etc.

(ii) Discrete Geometry –It is concerned with the relative position of simple geometric object, such as points, lines, triangles, circles etc.

(iii) Differential Geometry – It uses techniques of algebra and calculus for problem-solving. The various problems include general relativity in physics etc.

(iv) Euclidean Geometry – It is the study of plane and solid figures on the basis of axioms and theorems including points, lines, planes, angles, congruence, similarity, solid figures. It has a wide range of applications in Computer Science, Modern Mathematics problem solving, etc.

(v) Convex Geometry – It Includes convex shapes in Euclidean space using techniques of real analysis. It has application in optimization and functional analysis in number theory.

(vi) Topology –It is concerned with properties of space under continuous mapping. Its application includes consideration of compactness, completeness, continuity, filters, function spaces, grills, clusters and bunches, hyperspace topologies, initial and final structures, metric spaces, proximal continuity, proximity spaces, separation axioms, and uniform spaces.

Plane Geometry:

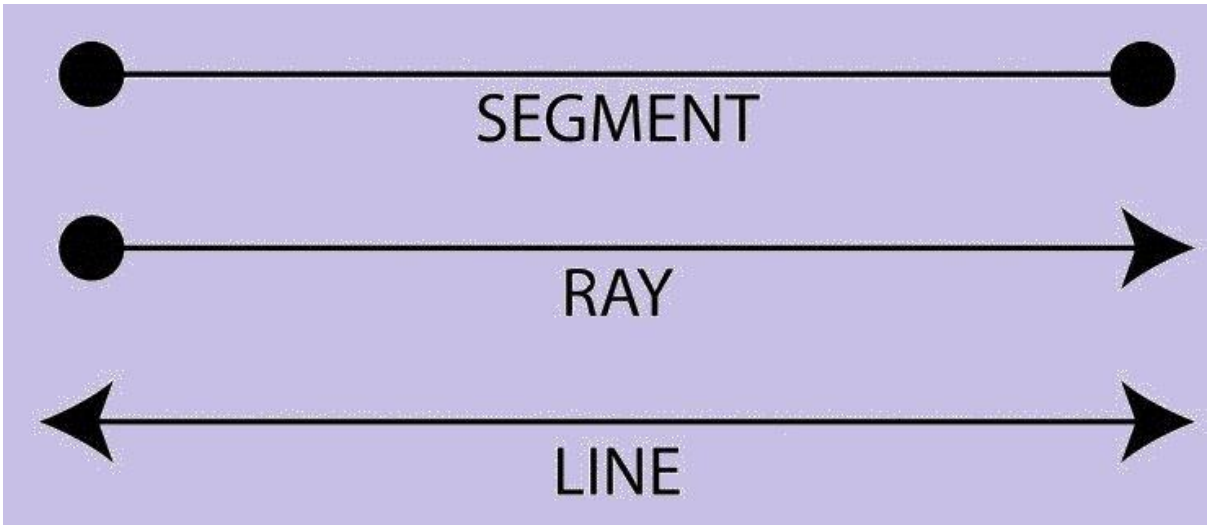
Plane Geometry deals with flat shapes which can be drawn on a piece of paper. These include lines, circles & triangles of two dimensions. Plane geometry is also known as a two-dimensional geometry. All the two-dimensional figures have only two measures such as length and breadth. It does not deal with the depth of the shapes. Some examples of plane figures are square, triangle, rectangle, circle, and so on.

Point :A precise location or place on a plane. Usually represented by a dot. A point is an exact position or location on a plane surface. It is important to understand that a point is not a thing, but a place. It is important to note that a point has no dimension rather it has the only position.

Line: The line is straight (no curves), having no thickness and extends in both directions without end (infinitely). It is important to note that it is the combination of infinite points together to form a line. In geometry, we have a horizontal line and vertical line which has x-axis and y-axis respectively.

Line Segment – If a line has a starting and an endpoint then it is called a Line Segment.

Ray – If a line has a starting point and has no endpoint is called Ray. Eg. Sun Rays



Angles:

In plane geometry, an angle is the figure formed by two rays, called the sides of the angle, sharing a common endpoint, called the vertex of the angle.

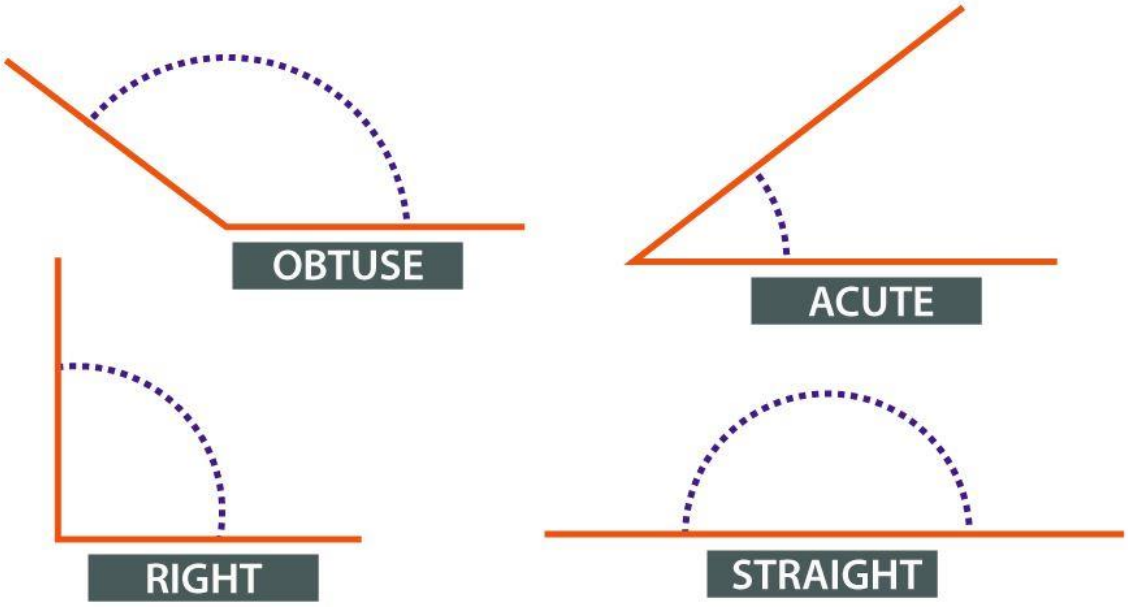
Types of Angle:

Acute Angle – An Acute angle (or Sharp angle) is an angle smaller than a right angle but greater than 0^0 i.e. it can range between $0^0 – 90^0$.

Obtuse Angle – An Obtuse angle is more than 90^0 but is less than 180 degrees.

Right Angle – An angle of 90^0.

Straight Angle – An angle of 180^0 is a straight angle. Such as angle formed by a straight line



Polygon

A plane figure that is bounded by a finite chain of straight line segment closing in a loop to form a closed polygonal chain or circuit.

The name ‘poly’ refers to multiple & gon is a polygon with n sides; for example, a triangle is a 3-gon polygon.

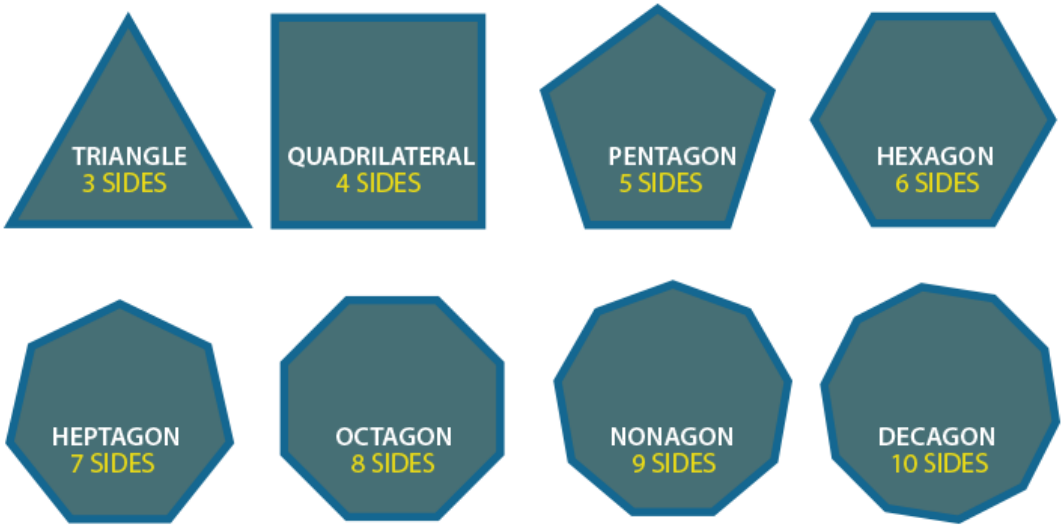
Sum of internal Angles of a polygon = $(n-2)\times 180^0$ Where n is number of sides.

Each internal angle of regular polygon = { $(n-2)\times 180^0$}÷ n

Sum of external Angles of a polygon = 360^0

Each external angle of regular polygon= {360^0}÷ n

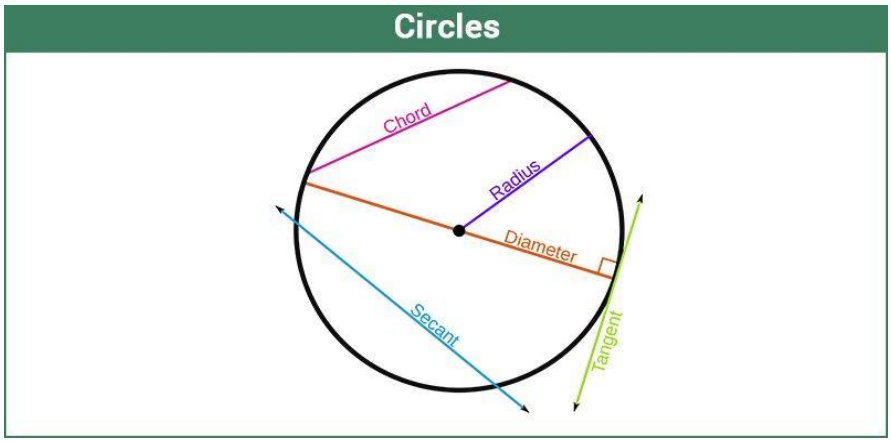
Types of Polygon



Polygon type	Definition & Property	Types
(i) Triangle –	A 3-sided polygon whose sum of internal angles always sums to 180 degrees.	• Equilateral Triangle – Has 3 equal sides and angles. • Isosceles triangle – Has 2 equal sides and angles. • Scalene triangle – Has all the 3 unequal sides and angles.
(ii)Quadrilateral	A 4-sided polygon with four edges and four vertices. Sum of internal angles is 360 degrees	• Square – Has 4 equal sides and vertices which are at right angles. • Rectangle – Has equal opposite sides and all angles are at right angles. • Parallelogram has two pairs of parallel sides.The opposite sides & opposite angles are equal in measure. • Rhombus – Has all the four sides to be of equal length. However, they do not have its internal angle to be 90 degrees • Trapezium – Has one pair of opposite sides to be parallel.
(iii) Pentagon	A plane figure with five straight sides and five angles	
(iv) Hexagon	A plane figure with six straight sides and six angles	
(v) Heptagon	A plane figure with seven sides and seven angles	

(vi) Octagon	A plane figure with eight straight sides and eight angles.
(vii) Nonagon	A plane figure with nine straight sides and nine angles.
(viii) Decagon	A plane figure with ten straight sides and ten angles.

Circle:A Circle is a simple closed shape. From a certain point called the centre, all points of a circle are of same consistent distance ie. the curve traced out by a point that moves so that its distance from the centre is constant.



Understanding Similarity and Congruence:

Similarity – Two figures are said to be similar if they have the same shape or have an equal angle but do not have the same size.

Congruence – Two figures are said to be Congruent if they have the same shape and size. Thus they are totally equal.

Solid Geometry:

Solid Geometry deals with 3-dimensional objects like cubes, prisms, cylinders & spheres. It deals with three-dimensions of the figure such as length, breadth and height. But some solid solids do not have faces (e.g. sphere). It is the study of three dimensions in the Euclidean space. The objects which are around us are obviously a three-dimensional. All the three-dimensional shapes are obtained from the rotation operation of 2D shapes. The important attributes of 3D shapes are faces, edges, and vertices. Let us discuss these terms in detail for different geometric shapes.

Edges:

An edge is defined as the line segment on the boundary that joins one vertex to the other vertex. It means that it joins one corner point to the other. It forms the skeleton of 3D shapes. In other words, it can be defined as the faces, that meets in the straight line is called edge. Following are the list of edges for the different solid shapes:

Solid Shapes	No. of. Edges
Triangular Prism	9
Cube	12
Rectangular prism	12
Pentagonal Prism	15
Hexagonal Prism	18
Triangular Pyramid	6
Square Pyramid	8
Pentagonal Pyramid	10
Hexagonal Pyramid	12

Faces: We know that all the geometric shapes are made up of flat surface called faces. It is a flat surface enclosed by the edges. For any three-dimensional shapes, the face should be a two-dimensional figure. The list of the number of faces for different solid shapes are given below:

Solid Shapes	No. of. Faces
Triangular Prism	5
Cube	6
Rectangular prism	6
Pentagonal Prism	7
Hexagonal Prism	8
Triangular Pyramid	4
Square Pyramid	5
Pentagonal Pyramid	6
Hexagonal Pyramid	7

Vertices: A vertex is defined as the point where the edges of the solid figure meet at each other. Or else, it can be said that, the point where the adjacent sides of the polygon meet. The vertex is the corners where the edges meet. The number of vertices for different solid shapes in geometry is as follows:

Solid Shapes	No. of. Vertices
Triangular Prism	6
Cube	8
Rectangular prism	8
Pentagonal Prism	10
Hexagonal Prism	12
Triangular Pyramid	4
Square Pyramid	5
Pentagonal Pyramid	6

Hexagonal Pyramid 7

Learning Resources in Teaching Mathematics

Chapter: 9

Various Aids in teaching Mathematics

Audio-Visual Aids in teaching mathematics

"Beauty attract to the person"

Popular saying on Audio- visual Aids:

The thing which I hear, I may forgot

The thing which I see, I may remembered

The thing which I do, I cannot forget

The use of Audio-Visual aids in teaching is not a fashion but is a matter belief and actual practice. That to say of ordinary visual aids e.g. charts. Graphs, map, models, etc. they are using films film-strips, epidiascope, tape-recorder, radio and television to make education valuable and worthwhile.

Audio-Visual Aids an Fit Well in:

a) Traditional system (from primary stage to higher secondary)

b) Basic system of education

c) Project method kindergarten Montessori etc.

Features of Audio-Visual Aids:

(i) Arouses interest, (ii) Modifies attitude, (iii) Claries concepts, (iv) Stimulates thinking, (v) Summarizes contents, (vi) Demonstrates knowledge and (vii) Concretizes knowledge.

We should use to Audio-visual packages to teach abstract and difficult oriented mathematical concepts, to enhance easy retention and high academic performance of our students. It will be effective and efficient in the classroom teaching-learning process.

PREPARATION OF TEACHING AIDS , MODELS & CHART

Mathematics is essentially a subject, where doing is more prominent than reading. That why a certain amount of equipment is indispensable in order to make even a start in this subject.

Teaching Learning material is the great tool to teach the students. It simplifies the subjects and makes teacher's presentation interesting and attractive. Students learn better by examples and T.L.M. In an innovative classroom just like smart class, there are many opportunities for a teacher to arrange many teaching learning materials online.

How to Prepare Mathematical Teaching Learning Materials in Class?

It is an easy and interesting activity to prepare mathematical T.L.M. and models in classroom. Teacher can provide it as a project for the groups of students. Teacher should facilitate the students to prepare the projects. Teacher can arrange the essential materials for students. When students learn how to prepare the T.L.M., they inspired to do such activities again and again.

Whiteboard as Powerful Teaching Learning Material:

Teachers use whiteboard in teaching the classroom. What are the smart and innovative ways to use whiteboard as effective teaching learning materials in mathematics teaching? Actually Smart and Active Teaching Learning Methodologies inspire the teachers to use more and more interactive teaching styles. These teaching methodologies ensure the student's participation in learning. Teacher can Facilitate the Students to use whiteboard to explain their problems, ideas and practices. When students use whiteboard they can learn better and they will be able to remember the concepts for longer time.

Smart and Innovative Teaching Learning Materials for Smart Classes:

In modern classrooms where we are planning to use internet and multimedia devices to make classroom learning easy, interactive and interesting .it is very nice to use internet based teaching learning material to make mathematics learning smart and innovative. What is the internet based learning materials? Internet provides several opportunities to make teaching interesting and interactive. Online Learning Games are the best example for this. There are many learning games for students to make mathematics learning easy and interesting. Smart School Class is very suitable for using these learning games better. Students learn better by games and interactive teaching techniques.

MODELS:

This 2D shapes class kit includes everything we need to teach students about 2D shapes. Teach them to sort, describe and name familiar two-dimensional shapes with a variety of resources, like tangrams, pattern blocks and attribute blocks. 'Toying with Tangrams', 'Developing Mathematics with Pattern Blocks' and 'Attribute Blocks – Developing Logic and Reason'.

The MTA Maths Kits have been designed to make life easier for the busy teacher, ensuring that a varied and appropriate set of resources is always to hand. Each kit includes popular maths resources which can be used to cover a number of activities

3D Shape:

Beginning geometry students get a hands-on introduction to solid forms with this tub of 3D plastic shapes. The 80 pieces include 8 each of 10 different geometric shapes from cones, spheres and cylinders to cubes, pyramids and more. The set is perfectly sized for desktop use. Geometric shapes set are a great tool to identify the different attributes of each shape, because students can see the shapes from each angle. Students can compare the shapes in the set with every day objects or find the number of faces and edges each shape has, to find out what makes each shape unique. The shapes can even be used for physics classes, to identify the shapes that easily roll, slide or can be stacked. By tracing the outline of the shapes, student can compare 2D and 3D shapes.

CHART: It helps in creating a suitable subject atmosphere in the class room and elucidating various points.

ICT IN TEACHING MATHEMATICS

Mathematics teachers were used in the study because the mathematics curriculum in particular emphasizes the use of ICT in the teaching and learning process.

It is quite clear that if technological development is to be achieved, proficiency in mathematics must be achieved. And for this to be achieved, all problems confronting mathematics teaching and learning in secondary schools that cause poor performance of students should be addressed. One way of doing this is through the use of information and communication technology. Mathematics is a useful tool in the society, more so in the present technology age. It plays important roles in the following areas; mathematics as core skill for life, mathematics as key to economic prosperity, and mathematics full of beauty and mathematics education.

In recent years, it seems there have been a sudden increase in the demand of ICT for teaching and learning of mathematics in our secondary schools such as computers, internet, overhead projectors, slide projectors, A-V materials, hand held calculator, printed materials, films motion pictures, sound and video recorders, improvised materials etc. The more effective these tools were used in teaching and learning of mathematics, the greater the understanding of the students for the subject. The resources include the use of spreadsheets, programs, dynamic geometry software, graph plotting software, video and internet research. All activities are complete with teachers' notes further ideas and the relevant software or files.

Objectives ICT in teaching Mathematics:

- The use of ICT in teaching and learning of mathematics improves students' performance and achievements.
- The use of ICT enhances teaching and learning of mathematics and improves students' problem solving skills.
- The use of ICT tools motivates and makes students interested in learning mathematics.

Chapter: 10

Mathematics Library and Mathematics Club.

Mathematics Library:

A school library is a library within a school where students, staff, and often, parents of a public or private school have access to a variety of resources. The goal of the school library media center is to ensure that all members of the school community have equitable access "to books and reading, to information, and to information technology. A school library media center "uses all types of media is automated, and utilizes the Internet as well as books for information gathering. School libraries are distinct from public libraries because they serve as "learner-oriented laboratories which support, extend, and individualize the school's curriculum... A school library serves as the center and coordinating agency for all material used in the school. Mathematics books, reference and related materials should be keep in separate in general library.

FEATURES OF MATHEMATICS LIBRARY:

1. Mathematics books, reference and related materials should be in keep in proper order.
2. Reference book of daily use should be available.
3. Materials should be based on needs of the students of teachers.
4. Reputed Authors books should be available.
5. INTERNET Facility.
6. Maximum number of copies should be available for every books.
7. Magazine related to mathematics.
8. Latest editions books/ Materials

Mathematics Club:

The meeting place where students of mathematics sit and discuss together is called mathematics club.

The club represents freedom and expression whereas the classroom represents conformity and repression . " M.C. Kown"

Aims of Mathematics club:

1. Touch with day by day invention.
2. To develop interest
3. To develop competitive spirit, Quiz, Debate etc.
4. To develop critical thinking and logical reasoning power.
5. To make the students mathematical minded.

Needs and importance of Mathematics club:

1. To learners to engage themselves in independent studies with freedom.
2. A less formal environment of study as compared to usual classroom teaching.
3. To the child for freedom of thoughts and free exchanges of different views.
4. It helps the child to become more social with mutual discussion in the group environment.
5. It helps the child to pass time more effectively with some purposeful activities.

Chapter:11

Co-curricular activities in Mathematics-Organizing Quiz Programme, Skill development in solving puzzles, riddles, magic & Using Mathematics as a game for recreation.

Chapter:12

Pedagogical analysis of teaching Mathematics:

Analysis: "The analysis of a given content material in any subject any topic carried out well in the spirit of science of teaching is known by the term Pedagogical Analysis of the

contents.” Pedagogical Analysis Pedagogy Science of Teaching Analysis Breaking down a Complex Systematic and Effective Teaching.

Need of Pedagogical Analysis :

1. Pedagogical analysis is selection of appropriate objectives and strategies in various instructional situations to access the level of actual teaching at the end.
2. A comprehensive vision of required tasks, strategies for realization of specific goals facilitates effective teaching. So pedagogical analysis offers enormous potential for improving the delivery of information in all form of education.
3. It involves various logical steps to arrive at logical inference.
4. It also helps the students to understand the concepts, principles and phenomena.
5. Effective Teaching or instructional objectives Methods and Learning Materials Evaluation Devices Content of the Subject.

The pedagogical analysis provides evidence as to the importance to students of such courses in developing subject knowledge for teaching as well as the value of face-to face programmes in supporting student learning across a range of learning outcomes of value to beginning teachers. Most of chapters in mathematics is linked with each other. So through the analysis students can understand easily and their proper concept will be develop and it will be satisfactory for students

Evaluation in Mathematics
Chapter:13

Evaluation

Evaluation = Measurement (Quantitative) + value judgment (Qualitative)

It is a continuous process and an important part of teaching – learning process.

Evaluation is the process of examining a program or process to determine what's working, what's not, and why. It determines the value of learning and training programs and acts as blueprints for judgment and improvement.

Formative Evaluation : – It is continuous evaluation in which unit tests class tests and assignments are the essential components. Formative evaluations are basically done on the fly. They permit the designers, learners, instructors, and managers to monitor how well the instructional goals and objectives are being met.

Formative evaluation is also useful in analyzing learning materials, student learning and achievements, and teacher effectiveness .Formative evaluation is primarily a building process which accumulates a series of components of new materials, skills, and problems into an ultimate meaningful whole. - Wally Guyot (1978)

Purposes of Formative Evaluation:

- To provide feedback for teachers to modify subsequent learning activities and experiences;
- To identify and remediate group or individual deficiencies.
- To move focus away from achieving grades and onto learning processes.
- To increase self efficacy and reduce the negative impact of extrinsic motivation.
- To improve students' metacognitive awareness of how they learn.
- "frequent, ongoing assessment allows both for fine-tuning of instruction and student focus on progress.

Characteristics of formative Evaluation:

According to Harlen and James (1997), formative Evaluation:

- It is essentially positive in intent, in that it is directed towards promoting learning; it is therefore part of teaching;
- It takes into account the progress of each individual, the effort put in and other aspects of learning which may be unspecified in the curriculum; in other words, it is not purely criterion-referenced;
- It has to take into account several instances in which certain skills and ideas are used and there will be inconsistencies as well as patterns in behaviour; such inconsistencies would be 'error' in summative evaluation, but in formative evaluation they provide diagnostic information;

- validity and usefulness are paramount in formative assessment and should take precedence over concerns for reliability;
- even more than assessment for other purposes, formative assessment requires that pupils have a central part in it; pupils have to be active in their own learning (teachers cannot learn for them) and unless they come to understand their strengths and weaknesses, and how they might deal with them, they will not make progress.

Summative Evaluation: It is type of half yearly / annual tests and external exams . Evaluation of refined product.A *summative evaluation* (sometimes referred to as external) is a method of judging the worth of a program at the end of the program activities (summation). The focus is on the outcome.

All assessments can be summative (i.e., have the potential to serve a summative function), but only some have the additional capability of serving formative functions. - Scriven (1967)

Characteristics of Summative Evaluation:

1) Authenticity

2)Reliability

3) Volume

4) Validity

5) Variety

Basic differences between the two types of Evaluation:

	Summative Evaluation	**Formative Evaluation**
When	At the end of a learning activity	During a learning activity
Goal	To make a decision	To improve learning
Feedback	Final judgement	Return to material
Frame of Reference	Sometimes normative (comparing each student against all others); sometimes criterion	Always criterion (evaluating students according to the same criteria)

Placement – The action of placing someone or something somewhere.

Placement evaluation, also referred to as pre-assessment or initial assessment, is conducted prior to instruction or intervention to establish a baseline from which individual student growth can be measured. This type of an assessment is used to know what the student's skill level is about the subject.

Placement evaluation is used to place students according to prior achievement or personal characteristics, at the most appropriate point in an instructional sequence, in a unique instructional strategy, or with a suitable teacher conducted through placement testing, i.e. the tests that colleges and universities use to assess college readiness and place students into their initial classes. Placement evaluation, also referred to as pre-assessment or initial assessment, is conducted prior to instruction or intervention to establish a baseline from which individual student growth can be measured. This type of an assessment is used to know what the student's skill level is about the subject. It helps the teacher to explain the material more efficiently. These assessments are not graded.

- **Diagnostic evaluation** – Diagnostic evaluation deals with the whole difficulties at the end that occurs during the learning process.

Jay McTighe and Ken O'Connor proposed seven practices to effective learning. One of them is about showing the criteria of the evaluation before the test. Another is about the importance of pre-assessment to know what the skill levels of a student are before giving instructions. Giving a lot of feedback and encouraging are other practices.

Chapter: 14

Principles for construction of objective, short answer and essay type tests and their comparative advantages.

Objective type test (very short answer) : The objective type test items are highly structured and require learners to supply (construct) or select response. These responses may consist of one or two words or from given options (multiple choice question). There can be only one correct answer they are highly objective in scoring.

Examples:

Q.1 What is the first positive integer?

Q.2 Solve the equation : 3x - 18 = 24 1)2 2)14 3)18 4)126

Advantage :

- Learners may solve answer quickly
- Answer can be solve by the given options
- It takes very less time to solve
- It can be calculated easily
- It is fit for the learners

Disadvantages:

- Guess work can be done
- It cannot be easier to construct.
- Each skills cannot be develop

Short answer (SA) test : The short answer (SA) test items require learners to supply (or construct) or select responses in one sentence or so.

Short answer items usually measure simple learning outcomes and mostly emphasize measure of recall of memorized information (response construction type) or identify information (selection type).

Examples:

Q.1The average age of a woman and her daughter is 46 years .The ratio of their present ages is 15:8 respectively .What is the daughter's age?

Q.2. The length of a rectangle exceeds its breadth by 7cms. If the perimeter of the rectangle is 126cm,then what will be the breadth of the rectangle?

Advantage : Short answer type items are being used widely for merits. They are very effective for measuring specific learning outcomes objectively and directly. They are easier to construct because these usually measure simple learning outcomes. They can use both types of test items: construct response (supply) and response selection. The main advantages of short answer type item can be summarized as follows:

- It can measure simplest learning outcomes.
- It is easier to construct.
- It can cover a wide sample of course materials. Constructed Response Items
- It can best be used in numerical questions requiring a number or symbol as an answer. Some of the short answer type test items can also measure knowledge and understanding
- Short answer type test items present well defined problems.

- It is simple to score with objectivity.

Disadvantages: Scholastic learning includes all those learning outcomes that are based on higher mental processes like application, analysis, synthesis, evaluation, problem-solving abilities, etc. Short answer (SA) test items suffer from many shortcomings so far as measurement of complex learning outcomes are concerned. Some of the disadvantages of short answer type items are that these do not provide:

- A more valid measure of real academic achievement of higher order learning outcomes.
- opportunities to relate facts, principles, thoughts etc.
- Sufficient opportunities for demonstration of logical progression of content, language flexibility of thought, coherence, interpretative ability etc.

Long answer (LA) : The long answer (LA test items require learners to select, organize and present answer in details form

It is known as long answer test. These tests make use of the ability to: (i) construct answer than merely identify interpretations and applications. (ii) to select organize and integrate ideas and express in original terms related to content, and (iii) provide freedom to learners to construct their own responses and thus encourage their originality and flexibility.

Examples:

Q.1.The digits of a two digit number differ by 2. If the digits are interchanged and resulting number is added to original number. We get 88. Find the original number.

Q.2. Prove that sum of angles of triangle is 180^0

Advantages:

- It can measure complex learning outcomes of higher order learning objectives and this use is not confined to recall of facts only.
- A question (especially extended response type) is its emphasis on the integration and application of thinking and problem-solving skills.
- Clear the complete concept to the learners
- Calculation skills will be more strong
- Develop the hypothesis skills.

Disadvantages:

- It takes much time to solve
- Learners can do many mistakes etc.

Chapter:15

Planning and preparation of Lesson plan & Unit plan

Learning how to plan effective lessons is one of the most important skills we will acquire. Having a good lesson plan is important for a whole host of reasons, not least in hopefully ensuring that learning will take place during the lesson. Not only is this because good planning results in lessons that are interesting, challenging and motivating for pupils, but also because planning is closely linked to the equally demanding (but often more overt) issue of effective classroom management. A good lesson plan, one that actively involves the class, helps to boost our confidence in the classroom and provides us with a sound basis for managing the class successfully. A good lesson plan goes a long way towards preventing classroom problems. Learning to plan good lessons needs work and effort, which takes time. This is because planning depends on our knowledge and understanding of a complex set of matters including: how pupils learn mathematics; the structure of the mathematics curriculum; the specific content, skills and concepts we are teaching; the prior knowledge of the pupils; ways of teaching mathematics; how lessons can be planned for maximum effectiveness. Devoting considerable time to planning is definitely worthwhile. It is a valuable investment for future years that, in the long term, reduces the demands of paperwork as planning becomes quicker and easier as experience grows.

Objective:

•Understand the relationship between the mathematics curriculum, a scheme of work, the choice of teaching strategies and your individual lesson plans.

• Select appropriate teaching strategies and mathematical tasks and resources (including ICT).

• Plan mathematics lessons and units of work, identifying clear objectives and content.

• Set appropriate and demanding expectations for pupil learning.

• Plan assessment opportunities.

• Work as part of a team in your planning and plan for out-of-school learning.

Importance of lesson Planning :

• Requires our to articulate what you think will happen in a lesson.

• Helps we to ensure that our lessons begin interestingly, maintain a good pace throughout, and have a satisfying ending.;

• Provides a basis for negotiation, discussion and evaluation;

• Creates a feeling of confidence.

• Provides a history of our thinking and development.

The lesson plans help us to:

• structure our lessons.

• build on previous lessons and learning.

• share the objectives of the lesson with pupils.

• assess pupil achievements so that we can take these into account in future lessons.

• develop effective 'assessment for learning', so pupils receive feedback that helps them to improve.

• make lessons more inclusive and address a range of needs.

• make better use of classroom support and learning assistants.

• make explicit the key teaching strategies.

• address the key questions we need to ask.

• set homework.

UNIT PLAN:

A unit plan is a comprehensive series of meaningful learning experiences built around a central theme or idea and organized in such a way as to result in appropriate behavioral change in pupils. It may extended from a minimum of 2-3days duration to one week or month or so depending on the content.

Characteristic of Unit plan:

1. It should fit into content
2. It should be reflect the abilities and interest of the students.
3. It should include provision of the variety of methods of teaching.
4. It should contain problem scope for evaluation.
5. It should provide necessary resources.

Need a Unit Plan:

Unit planning provides us with a sense of direction and organization that again helps us and the class to achieve significant academic gains within a particular time period

1. **A unit plan forces to make difficult decisions about what to teach and how to teach it**. After taking the time to develop a unit plan, we are less likely to be side-tracked by objectives,
lessons, or activities that do not advance our ultimate quest for academic achievement.

2.A unit plan keeps on pace to reach our unit (and ultimately long-term) goals. our unit plan, which should be referred to with almost daily frequency, is our point of reference when we
ask ourself, "Given where I want to be in two or four or six weeks, am I where I need to be now? Am I spending too much time on certain skills and concepts given the other skills and concepts that must be included in these X weeks, or X days?" Given the limited number of weeks, days, and lessons in a unit, each moment becomes more precious, forcing to pace our self appropriately in order to meet our end goals.

3.A unit plan provides an opportunity to stimulate student interest through overarching content that is relevant to students. When we design our unit plan, consider what content will engage our students given their interests and backgrounds. As Jere Brophy indicates in Tomorrow's Teachers, "whether in textbooks or in teacher-led instruction, information is easier to learn to the extent that it is coherent (i.e., a sequence of ideas or events makes sense and the
relationships among ideas are made apparent).

Chapter: 16

Preparation of Achievement Test in Mathematics-planning, preparation tryout and evaluation.

Achievement Test

Meaning:- The objective of evaluation is to provide a clear reliable and valid description about the pupil's achievement.

Types:- Teacher made tests (written , oral, objective, essay type.)

Standardized test:- Uniformity of procedure in scoring, administering and interpreting the test results. (To diagnose learning difficulties evaluation of curriculum)

Planning the test:

1) What is to be measured?
2) What content areas should be included ?
3) What types of test items are to be included?

Therefore the 1st step includes three major considerations.

1) Determining the objectives of testing.
2) Preparing test specifications in teaching and learning process
3) Selecting appropriate item types.

Preparing test specification :

Weightage for the instructional objectives:

Knowledge – 30% Understanding - 30% Application – 30% Skill- 10%

Weightage to different topics = (total no. of items (marks)/ total no. of pages in the book)

×no. of pages in topics

Selecting appropriate items types:

To decided appropriate item types. (objective type, essay type etc.)

SU – SUPPLY TYPE

MC – MULTIPLE CHOICE

MT – MATCHING TYPE ITEMS

TF – TRUE FALSE ITEMS.

Preparing the test :

Preparing the test items : Learning out come to be measured , all types of instructional objectives and the whole content area,

The test item should be free from ambiguity i.e should be clear

Examlpe: poor items- How many sides in polygon?

Better – How many sides in Triangle?

Appropriate difficulty level free from Technical error.

2.Preparing instruction for the test: Direction (time, marks, steps etc)

3. Preparing the scoring key : Marking Scheme

Try out of the test (validity, Reliability, usability) of test.

a) **Functions**- Administration of the test: Sitting arrangement, light, infrastructures, ventilation

b) **Scoring of The Test**

Score = R – (W/N-1)
R – No. of right response
W - No. of wrong response
N –No. of alternative

Evaluating the test: Quality of the test, quality of the responses

Functions:-

a)Item Analysis – (appropriate difficulty level) Multiple choice type items are effective.

Item analysis data also help us

Efficient class discussion

The remedial works.

To increase skill in test construction.

To improve class room discussion.

Item Analysis Procedure :–

27% from highest, 27% from lowest out of 60 (16 – h, 16 – L)

Item difficulty: (R/T)× 100

R – No. of students response correct (right)

T – No. of students tried the item

Out of 32 students , R – 20, T – 30

Find. I.D =(20/30) × 100.

If value comes < 25% Too difficult, 25 %<I.D<75% difficult > 75% - Too easy item.

Discriminating power = $(R_U – R_L) /T/2$

R_U – No. of students response the right answer from upper group.

R_L – No. of students response the right answer from lower group

T – Total no. of students

Example: If R_U = 15 , R_L = 5 T= 32

Discriminating power (D.P) = (15-5)/16 = .63

.63 – average discriminating power

If $R_U = R_L$ i.e 0 discriminating power

If R_U = All correct, R_L = all incorrect

i.e 1 it means maximum positive discriminating power.

b) Determining Validity of the test

c) Determining Reliability of the test

Chapter: 17
Diagnostic test and remedial measures, Action Research in Mathematics.

DIAGNOSTIC TEST

Mathematics teacher applies diagnostic test to diagnose the particular strength and weakness of the students. It is qualitative.
How much water in bottle? Quantity of water comes in achievement test, while about quality why the bottle is empty or full comes in diagnostic test.

Diagnostic testing can be both formal and informal. Formal diagnostic testing includes standardized tests that can be used to assess particular skills, giving objective data on skill levels. However, the validity of such tests can be debated, and there is some concern about test bias. Additionally, standardized tests may assess more or fewer skills than those that will occur during instruction. The formal approach to diagnostic testing can be implemented within a classroom, a department, or within a school. It can also occur within a school district, state, or nation.

Informal diagnostic testing approaches can provide more flexibility, such as one-on-one questioning or small-group testing; but they still must follow the principles of diagnostic testing, meaning that they must assess only what is slated to be taught in the classroom and cover all concepts and skills.

Types of Diagnostic Test:

1. **Educational Diagnostic Test :** Educational diagnostic testing is a form of assessment that occurs before instruction begins. The purpose of administering diagnostic tests is to try to determine what students already know about the concepts and skills to be covered by instruction. The tests are not graded. The tests can determine if differentiated instruction is need, and discover students' preferred learning styles as well as their strengths, weaknesses, and misconceptions. Diagnostic tests are designed to closely follow what will be asked on a summative assessment and can be used to predict how well students will perform on high-stakes tests used to meet No Child Left Behind guidelines and state standards. In this respect, they can be considered a combination of both summative and formative assessments.

2. **Physical or clinical Diagnostic Tests.** A medical **test** is a kind of medical procedure performed to detect, diagnose, or monitor diseases, disease processes, susceptibility, and determine a course of treatment. It refers to hearing, vision and other things which is affect to the child for learning.

Characteristic of Diagnostic Test:

1. To find out weakness or deficiency.
2. To help in planning and organizing remedial teaching.
3. It adopts objective type tests only.

Functions of Diagnostic Test :

1.Classification of Aptitude or musical level ,vocational level, intellectual level.
2. Assessment of specific abilities with regard to – level of abnormality, level of depression and anxiety & level of adjustment.
3. Remediation – Special education for handicapped, remedial teaching for learning weakness, counseling for mental ailment, clinical treatment for physical ailment.
4. Etiology – Study of diagnosis.

METHODS:-

1) Testing method:- Educational testing, psychological testing & clinical testing .
2) Diagnostic mathematical skills test :- (mathematical operations, decimal, previous no. next no. etc.)

Steps for Construction of Diagnostic Test:

- Formulate the objectives
- Analysis the content into subtopics and its elements.
- Prepare the final draft
- Prepare manual of test
- Remedial devices and measures

Diagnostic Testing in the Classroom:

Diagnostic testing must be aligned with predetermined learning objectives and should be built into the regular classroom routine. The assessments should be relatively short, valid, and free from bias. In order to accurately use diagnostic testing, instructors must be willing to modify course content and their teaching methods based on the information they receive from the assessments. This could mean covering subjects and concepts assumed to be already mastered, or not covering concepts that were originally planned if the skills and concepts have already been mastered.

Remedial Measures

Remedial instruction is the process of providing help to students who are experiencing difficulties so that they can understand and master the concept with which they are struggling. In math, each concept is the foundation for new learning, and when a student has not mastered one concept they are unable to move on to the next concept. In this case, remediation helps to get the student back on track so they can continue their learning on the math continuum.

Teaching remedial math means that we will be working with students who might be missing foundational skills that are required for learning higher level concepts. Whether because of a learning difficulty or another situation that impacted their learning, these are students, for example, who may not have mastered adding one digit numbers and are now being asked to add two-digit numbers with regrouping.

Identify the Issue:

We can't just throw a bunch of ideas or strategies at the problem and hope something works for the student. We must first identify the specific area in which the student is struggling so we can begin to create a plan and strategies for remediation. Using the example of the student struggling with one digit addition - you need to figure out if he hasn't mastered one digit addition because he has spatial difficulties that are interfering with his ability to regroup. Or has he even mastered counting? You would first need to identify what is causing the difficulty before you can begin working to help the student learn.

The following difficulties among the students.

1. More than half of the students incorrectly measure the units.
2. Most of students do mistake in calculation part in maths.
3. Often get confusion between the areas and volumes in the word problem and between square and cubic units.
4. Poor understanding in the algebraic identities.
5. Students have find difficulties in solving algebraic word problems.
6. Difficulties in trigonometrically ratios for the beginners of trigonometric concept.
7. Understanding in trigonometry identities.
8. Some students are memorizing the concept without understanding.

Use a Variety of Approaches:

Math education lends itself especially well to approaching concepts from a variety of different angles. Just think of the wide variety of ways students can solve a simple addition problem. They can draw a picture of objects and count them, use manipulative, write a story, count on a number line, or count in their head. When teaching remedial math, it is important to explore as many approaches to learning as possible to help your students.

For example, we may have to teach a student who is struggling with comparing fractions. For some reason, this student may just not grasp this concept, no matter how hard they try. Sitting down with the student, you discover that the student is just staring at the fractions trying to will the answer into existence.

Conclusion:

In the present 21st century the explosion of technologies uplift the world into the heaven in the sky. It leads to globalization. We require powerful brainy citizens for this competitive world. Education can give tremendous boost to these citizens in the global society. Education should not only reflect the needs of the society but also excellence. Every effort should make to adopt our educational system today’s changing economic and social realities of the scientific world. No branch of science is complete without mathematics. Mathematical understanding and reasoning are essential components of success in all walks of life.

Diagnostic Testing which is the most important part of the teaching-learning process. It implies a detailed study of learning difficulties. Its aim is to analyze, not to assess. The nature

and purpose of Diagnostic Testing is to identify the areas of difficulties where the learner commits errors.

The stages of diagnostic testing are:

i) Identifying the students who need help.

ii) Locating the error/learning difficulties.

iii) Discovering the causal factors.

After locating the area where the difficulty lies, as a teacher we will devise some strategy to remove problems in learning and the causes due to which the learner has faced the difficulties. The strategy used by you to remove the weakness of the learner is known as remedial teaching. Diagnostic Testing leads to remedial teaching in which you have to prepare instructional material for quality learning, adopting different methodologies as per needs of the individual or a particular group.